Test	Condition	Example	Exercise	
Integral	$a_n = f(n), f\begin{cases}\text{positive}\\ \text{continuous}\\ \text{decreasing}\end{cases}$ $\int_1^\infty f(x)dx\ (\text{con}) \Rightarrow \sum_{n=1}^\infty a_n\ (\text{con})$ $\int_1^\infty f(x)dx\ (\text{div}) \Rightarrow \sum_{n=1}^\infty a_n\ (\text{div})$	$\sum_{n=1}^\infty \frac{\ln n}{n} \Rightarrow \int_1^\infty \frac{\ln x}{x}dx \begin{cases} u = \ln x\\ x = 1, u = 0\\ du = \frac{1}{x}dx\end{cases} \Rightarrow \int_1^\infty \frac{\ln x}{x}dx = \int_0^\infty udu = \frac{u^2}{2}\Big\vert_0^\infty\ (\text{div})$ $\sum_{n=1}^\infty \frac{n}{2n^2+1} \Rightarrow \int_1^\infty \frac{x}{2x^2+1}dx \begin{cases} u = 2x^2+1\\ x = 1, u = 3\\ du = 4x\end{cases} \Rightarrow \int_1^\infty \frac{x}{2x^2+1}dx = \frac{1}{4}\int_3^\infty \frac{du}{u}$ $= \frac{1}{4}\ln u\Big\vert_3^\infty\ (\text{div})$ 特点: $\begin{cases}\frac{f'(x)}{f(x)} & 特别注意\ (\ln x)' = \frac{1}{x}\\ f'(x)\cdot f(x) & 特别注意\ \mathrm{e}^x = e^x\end{cases}$	$\sum_{n=1}^\infty ne^{-n^2}$	$u = e^{-x^2}, x = 1\ \ u = \frac{1}{e}, du = -2xe^{-x^2}$ $\int_1^\infty xe^{-x^2}dx = \frac{-1}{2}\int_{1/e}^0 du = \frac{-1}{2}u\Big\vert_{1/e}^0\ (\text{con})$
			$\sum_{n=2}^\infty \frac{1}{n\ln n}$	$u = \ln x, x = 2\ \ u = \ln 2, du = 1/x$ $\int_2^\infty \frac{1}{x\ln x}dx = \int_2^\infty \frac{\frac{1}{x}dx}{\ln x} = \int_2^\infty \frac{d\ln x}{\ln x}$ $= \int_{\ln 2}^\infty \frac{du}{u} = \ln u\vert_{\ln 2}^0\ (\text{div})$
Direct	$0 < a_n \le b_n$ $\sum_{n=1}^\infty b_n\ (\text{con}) \Rightarrow \sum_{n=1}^\infty a_n\ (\text{con})$ $0 < b_n \le a_n$ $\sum_{n=1}^\infty b_n\ (\text{div}) \Rightarrow \sum_{n=1}^\infty a_n\ (\text{div})$	$\sum_{n=2}^\infty \frac{n}{n^2-3},\quad \frac{n}{n^2-3} > \frac{n}{n^2} = \frac{1}{n},\quad \frac{1}{n}(\text{div})$ 特点：分式函数 $\frac{g(x)}{f(x)\pm 常数}$	$\sum_{n=1}^\infty \frac{1}{n!}$	当$n \ge 4$ 时 $\sum_{n=1}^\infty \frac{1}{n!} < \sum_{n=1}^\infty \frac{1}{n^2} \Rightarrow (\text{con})$
		$\sum_{n=2}^\infty \frac{\sin^2 n}{\sqrt{n^3}+1},\quad \frac{\sin^2 n}{\sqrt{n^3}+1} \le \frac{1}{\sqrt{n^3}+1} < \frac{1}{\sqrt{n^3}} \xRightarrow{p=\frac{3}{2}>1} (\text{con})$	$\sum_{n=1}^\infty \frac{1}{\sqrt{n}+2}$	$\sum_{n=1}^\infty \frac{1}{\sqrt{n}+2} < \sum_{n=1}^\infty \frac{1}{\sqrt{n}} \Rightarrow (\text{div})$
Limit	$a_n > 0, b_n > 0\ and\ \lim_{n\to\infty}\frac{a_n}{b_n} = L$ $\sum_{n=1}^\infty b_n\ (\text{con}) \Rightarrow \sum_{n=1}^\infty a_n\ (\text{con})$ $\sum_{n=1}^\infty b_n\ (\text{div}) \Rightarrow \sum_{n=1}^\infty a_n\ (\text{div})$	$\sum_{n=2}^\infty \frac{1}{\sqrt{n^2+4}},\quad a_n = \frac{1}{\sqrt{n^2+4}}, b_n = \frac{1}{\sqrt{n^2}}, \lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{n}{\sqrt{n^2+4}} = 1, \sum_{n=2}^\infty \frac{1}{\sqrt{n^2}}(\text{div})$	$\sum_{n=2}^\infty \frac{2n}{2^n\sqrt{n^2+1}}$	$a_n = \frac{2n}{2^n\sqrt{n^2+1}}, b_n = \frac{1}{2^n} \Rightarrow (\text{con})$
		$\sum_{n=1}^\infty \frac{3^n}{n!},\quad \frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{(n+1)!}\frac{n!}{3^n} = \frac{3}{n+1}, \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = 0 < 1(\text{con})$	$\sum_{n=1}^\infty \sin\frac{1}{n}$	$a_n = \sin\frac{1}{n}, b_n = \frac{1}{n} \Rightarrow (\text{div})$
Ratio	$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} < 1 \Rightarrow \sum a_n\ (\text{con})$ $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} > 1 \Rightarrow \sum a_n\ (\text{div})$ $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = 1 \Rightarrow \sum a_n\ (??)$	$\sum_{n=1}^\infty \frac{n!}{n^n},\quad \frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(n+1)^{n+1}}\frac{n^n}{n!} = \frac{(n+1)\cdot n!}{(n+1)(n+1)^n}\frac{n^n}{n!} = \left(\frac{n}{n+1}\right)^n,$ $\Rightarrow \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{1}{e} < 1(\text{con})$	$\sum_{n=1}^\infty \frac{3^n}{n!}$	$\frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{(n+1)!}\frac{n!}{3^n} = \frac{3}{n+1}, \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = 0$ $< 1(\text{con})$
		$\sum_{n=1}^\infty \frac{3^n}{2^n-1},\quad \frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{2^{n+1}-1}\frac{2^n-1}{3^n} = \frac{3(2^n-1)}{2^{n+1}-1} = \frac{3\left(1-\frac{1}{2^n}\right)}{2-\frac{1}{2^n}}, \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{3}{2}$ $> 1(\text{div})$	$\sum_{n=1}^\infty \frac{\cos^n x}{2^n}$	$\frac{a_{n+1}}{a_n} = \frac{\cos^{n+1}x}{2^{n+1}}\frac{2^n}{\cos^n x} = \frac{\cos x}{2} < 1(\text{con})$

$\ln n, n, n^2, n^3 \ldots;\qquad 2^n, e^n, 3^n \ldots;\qquad n!;\qquad n^n$

Series	Property	Example	Exercise	
Geometric	$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}, if\ \lvert r \rvert < 1$	$\sum_{n=1}^{\infty}\frac{2^{n+1}}{3^n} = \sum_{n=1}^{\infty} 2\cdot\frac{2}{3}\cdot\frac{2^{n-1}}{3^{n-1}} \Rightarrow \begin{cases} a = 4/3 \\ r = 2/3 \end{cases} \Rightarrow \sum_{n=1}^{\infty}\frac{2^{n+1}}{3^n} = \frac{4/3}{1-2/3} = 4$ $if\ S_n = \left(\frac{3^{n-1}}{(4+n)^{20}}\right)\left(\frac{(7+n)^{20}}{3^n}\right), to\ what\ number\ does\ the$ $sequence\ \{S_n\}\ converge?\quad S_n = \frac{1}{3}\left(\frac{3}{4+n}+1\right)^{20}, \lim_{n\to\infty} S_n = \frac{1}{3}$	$\sum_{n=1}^{\infty}\frac{3^{n+1}}{5^n}$	$= \sum_{n=1}^{\infty}\frac{9}{5}\cdot\left(\frac{3}{5}\right)^{n-1} \Rightarrow \begin{cases} a = 9/5 \\ r = 3/5 \end{cases} \Rightarrow \sum_{n=1}^{\infty}\frac{3^{n+1}}{5^n} = \frac{9/5}{1-3/5}$ $= \frac{9}{2}$
			$\sum_{n=1}^{\infty}(\tan 1)^n$	$\begin{cases} a = \tan 1 \\ r = \tan 1 > 1 \end{cases} \Rightarrow (\text{div})$